

Laser cooling all the way down to Fermi superfluid

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Numerical simulations for realistic experimental parameters demonstrate that laser cooling on the attractive side of the Feshbach resonance can drive fermions much below the superfluid transition. For the assumed set of experimental parameters the transition takes place at $0.35T_F$, and laser cooling can drive the system down to at least $0.085T_F$ in a time of a few seconds. Superfluid growth is self-consistently included in simulations.

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Fermi superfluids are at the forefront of both experimental and theoretical activity in the physics of ultracold dilute atomic gases. In addition to being a challenge in their own right, they provide promising model systems to test the theory of fermionic superfluidity, or the Bardeen-Cooper-Schrieffer (BCS) theory, in the regime of strong interactions, when the size of Cooper pairs becomes comparable, or even less than the average interparticle distance. In this regime excitations of the system develop a pseudogap indicating that Cooper pairs are quasi bound states of pairs of fermions [1]. This is also the limit that seems to be a key to understanding the physics of high T_c superconductors, where pseudogap effects have been observed experimentally. The cold atoms provide a model system of high flexibility thanks to application of the Feshbach resonance technique [2]. The Feshbach resonance between an open and closed channel allows one to manipulate an effective s-wave scattering length between pairs of atoms with opposite “spin”. This technique allows one to drive the system through the BCS-BEC crossover, when the scattering length diverges and changes sign from negative (attractive) to positive (repulsive). At the same time the state of the system continuously evolves from the BCS superfluid of weakly correlated Cooper pairs, through the pseudogap regime, and then the unitarity limit, when the scattering length diverges, to the Bose-Einstein condensate of biatomic molecules.

The main problem in the experiments on atomic Fermi superfluids is how to cool the system sufficiently deep into the quantum degenerate regime to reach the superfluidity. Unfortunately the standard evaporative cooling stops to work in the degenerate regime, but before the superfluidity is achieved [3]. One of the reasons may be creation of holes deep in the Fermi sea by collisions with the buffer gas [4]. However, this problem can be circumvented in an ingenious way thanks, again, to the Feshbach technique

[5]. The idea is to obtain first a molecular condensate on the BEC side of the Feshbach resonance, and then continuously transform it into BCS superfluid by an adiabatic passage to the other side of the resonance. This adiabatic transition reduces the temperature sufficiently to reach the superfluid state. By now, several successful experiments with molecular condensates were performed, and paved a way to first successful experiments with Fermi superfluids [6,7]. The Innsbruck experiment [7] is the first to report direct signatures of Fermi superfluidity in quasi-particle excitation spectrum, in excellent agreement with the theory of Refs. [8].

The purpose of this Letter is twofold. First, we demonstrate that it is possible to laser cool a Fermi gas down to the superfluid BCS state on the attractive (BCS) side of the Feshbach resonance. This is challenging because, so far, the standard laser cooling has not even allowed for observing quantum degeneracy. However, theoretical studies of laser cooling show in particular that it is possible to reach the degeneracy in Fermi systems [9]. Laser cooling offers an alternative route towards BCS superfluid directly on the negative side of the Feshbach resonance. Moreover, it opens a possibility to cool the BCS superfluid prepared via the “standard” method down to even lower temperatures. The second goal of this Letter is to develop for the first time fully self-consistent quantum kinetic theory of BCS state creation via laser cooling ¹. The results are very promising: despite the fact that both BCS formation and normal mean field effects shift the energies of quasiparticle excitations with respect to the equally spaced harmonic oscillator levels, the cooling remains efficient at least down to the temperature $T \simeq 0.085T_F$. This temperature is well below the superfluid transition temperature which for the assumed reasonable experimental parameters is close to $0.35T_F$. Optimization of the cooling protocol could significantly reduce this temperature.

We consider two species of fermions with “spin up and down”, molecules which are their bound states, and fermions in an excited state. Populations of the spin up and down fermions are the same. We assume that mutual s-wave interaction between these two populations leads to thermalisation on a timescale, which is faster than the rate of laser cooling. The system remains in

¹similar to self-consistent theories of the BEC growth [10]

a quasi-equilibrium state with slowly decreasing temperature. This equilibrium state is described by the generalised weak coupling BCS theory, or boson-fermion model [11] which includes Bose-Einstein condensate of the molecules. Structure of the ground state and its excitations follows from the set of Bogoliubov-de Gennes equations together with self-consistent definitions of the gap function and the mean field potential. Laser cooling of fermions was described in detail in Ref. [9]. Coherent laser excites atoms from, say, the spin down ground state to the excited state and at the same time spontaneous emission brings them back to the spin down ground state. Frequencies of the laser assure that the generalized Raman cooling takes place [12]. In this Letter we take into account interactions between fermions and describe the cooling process in terms of instantaneous Bogoliubov quasiparticles, whose eigenfunctions and eigenenergies are self-consistently updated during the evolution. We work in the *festina lente* limit to avoid reabsorption effects [13], and also employ spherical symmetry and ergodic approximations (see also [10]).

The boson-fermion model [11] is defined by the Hamiltonian

$$\hat{H} = \int d^3r \left[\sum_{a=+, -, e} \hat{\psi}_a^\dagger \mathcal{H}_1 \hat{\psi}_a + g_0 \hat{\psi}_+^\dagger \hat{\psi}_-^\dagger \hat{\psi}_- \hat{\psi}_+ + \lambda \hat{\psi}_m^\dagger \hat{\psi}_+ \hat{\psi}_- + \text{h.c.} + (2\nu - 2\mu) \hat{\psi}_m^\dagger \hat{\psi}_m \right]. \quad (1)$$

The fields $\hat{\psi}_+$ and $\hat{\psi}_-$ describe the fermions with spin up and down respectively, $\hat{\psi}_e$ corresponds to the fermions in the excited state, and $\hat{\psi}_m$ is the bosonic molecular field. Here $\mathcal{H}_1 = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) - \mu$ is a single particle Hamiltonian, m is atomic mass, $V(\vec{r})$ is a harmonic trap potential, μ is chemical potential, $g_0 = \frac{4\pi\hbar^2 a_0}{m}$ is bare interaction strength with a negative bare s -wave scattering length a_0 , λ is a coupling between pairs of atoms and molecules, and ν is detuning from the Feshbach resonance.

In the mean field approximation, which closely follows the BCS theory, the quartic interaction term in the Hamiltonian (1) is made quadratic by replacing all products of pairs of operators with their averages. The averages are the mean field potential $W(\vec{r}) = g_0 \langle \hat{\psi}_\pm^\dagger(\vec{r}) \hat{\psi}_\pm(\vec{r}) \rangle$, and the anomalous potential of the pairing field $P(\vec{r}) = \langle \hat{\psi}_+(\vec{r}) \hat{\psi}_-(\vec{r}) \rangle$, which is mixing fermions with spin up and down. The molecular field $\hat{\phi}_m$ is replaced by the amplitude of molecular condensate $\langle \hat{\phi}_m \rangle$. After elimination of the molecular condensate one obtains linear equations for the spin up and down fermions

$$i\hbar \frac{d}{dt} \hat{\psi}_\pm = \mathcal{H}_1 \hat{\psi}_\pm + W(\vec{r}) \hat{\psi}_\pm \mp g P(\vec{r}) \hat{\psi}_\mp^\dagger, \quad (2)$$

with an effective coupling constant $g = g_0 + \frac{\lambda^2}{2(\mu - \nu)}$.

These equations mix $\hat{\psi}$ and $\hat{\psi}^\dagger$, but can be “diagonalized” by the Bogoliubov transformation

$$\hat{\psi}_+(\vec{r}) = \sum_m \hat{b}_{m,+} u_m(\vec{r}) - \hat{b}_{m,-}^\dagger v_m^*(\vec{r}), \quad (3)$$

$$\hat{\psi}_-(\vec{r}) = \sum_m \hat{b}_{m,-} u_m(\vec{r}) + \hat{b}_{m,+}^\dagger v_m^*(\vec{r}), \quad (4)$$

with fermionic quasiparticle annihilation operators $\hat{b}_{m,\pm}$. The Bogoliubov modes (u_m, v_m) fulfill the Bogoliubov-de Gennes equations

$$\begin{aligned} \omega_m u_m &= + \mathcal{H}_1 u_m + W u_m + \Delta v_m, \\ \omega_m v_m &= - \mathcal{H}_0 v_m - W v_m + \Delta^* u_m, \end{aligned} \quad (5)$$

with positive energies ω_m of quasiparticle excitations. Here $\Delta = -gP$ is the quasiparticle gap function. In a thermal state with inverse temperature β , the average occupation numbers of quasiparticle states are given by Fermi-Dirac distribution $N_m = \langle \hat{b}_{m,\pm}^\dagger \hat{b}_{m,\pm} \rangle = (1 + \exp(\beta\omega_m))^{-1}$. Equations (5) are solved together with the self-consistency conditions

$$\Delta(\vec{r}) = \left[-g \sum_m (1 - 2N_m) u_m(\vec{r}) v_m^*(\vec{r}) \right]_{\text{reg}}, \quad (6)$$

$$W(\vec{r}) = g_0 \sum_m (1 - N_m) |v_m(\vec{r})|^2 + N_m |u_m(\vec{r})|^2, \quad (7)$$

by successive iterations with the chemical potential μ adjusted to keep the average number of atoms constant. The ultraviolet divergence in Eq.(6) is regularized using the quickly convergent method of Ref. [14].

Excitation of atoms from the spin down state to the excited state is described by the Hamiltonian

$$\hat{H}_{\text{las}} = \int d^3r \left[\frac{1}{2} \Omega e^{i\vec{k}_L \vec{r}} \hat{\psi}_e^\dagger(\vec{r}) \hat{\psi}_-(\vec{r}) + \text{h.c.} - \delta \hat{\psi}_e^\dagger \hat{\psi}_e \right], \quad (8)$$

driving coherent oscillations with Rabi frequency Ω and laser detuning δ . The excitation is accompanied by the spontaneous emission, which after rotating wave approximation is described by a superoperator

$$\mathcal{L} \hat{\rho} = \gamma \sum_{mk} U_{mk} \mathcal{D}[\hat{b}_{m,-}^\dagger \hat{e}_k] \hat{\rho} + V_{mk} \mathcal{D}[\hat{b}_{m,+} \hat{e}_k] \hat{\rho}, \quad (9)$$

with a spontaneous emission rate of γ . Here the Lindblad superoperator is $\mathcal{D}[\hat{A}] \hat{\rho} = \hat{A} \hat{\rho} \hat{A}^\dagger - \frac{1}{2} \hat{\rho} \hat{A}^\dagger \hat{A} - \frac{1}{2} \hat{A}^\dagger \hat{A} \hat{\rho}$, and the matrix elements are e.g. $U_{mk} = \int d\Omega_k \mathcal{W}(\Omega_k) |u_{mk}(\vec{k})|^2$ with the spontaneous emission pattern $\mathcal{W}(\Omega_k)$ and the generalized Frank-Condon factors $u_{mk}(\vec{k}) = \int d^3r e^{i\vec{k} \vec{r}} w_k^*(\vec{r}) u_m(\vec{r})$. Here $w_k(\vec{r})$ is the k -th eigenstate of the harmonic oscillator and \hat{e}_k an annihilation operator of an excited atom in this state.

Adiabatic elimination of the excited state, similar as in Ref. [15], leads to the kinetic equations for the occupation numbers taking into account the effects of Fermi statistics

$$\begin{aligned}\frac{dN_{m,-}}{dt} &= \sum_n \Gamma_{m \leftarrow n}^{(-)} (1 - N_{m,-}) N_{n,-} - (n \leftrightarrow m) + \\ &\quad C_{mn} (1 - N_{m,-}) (1 - N_{n,+}) - A_{nm} N_{m,-} N_{n,+}, \\ \frac{dN_{m,+}}{dt} &= \sum_n \Gamma_{m \leftarrow n}^{(+)} (1 - N_{m,+}) N_{n,+} - (n \leftrightarrow m) + \\ &\quad C_{nm} (1 - N_{m,+}) (1 - N_{n,-}) - A_{mn} N_{m,+} N_{n,-}.\end{aligned}$$

They describe relaxation of \pm quasiparticles ($\Gamma_{m \leftarrow n}^{(\pm)}$), and creation/annihilation of pairs of $+$ and $-$ quasiparticles (C_{nm} and A_{nm}). The transition rates are e.g.

$$\Gamma_{m \leftarrow n}^{(-)} = \frac{\Omega^2}{2\gamma} \sum_k \frac{\gamma^2 U_{mk} |u_{nk}(\vec{k}_L)|^2}{(\delta - \omega_k^e + \omega_n)^2 + \gamma_k^2}. \quad (10)$$

Here ω_k^e is the energy of the k -th harmonic oscillator state and γ_k is approximate spontaneous decay rate of an excited atom in this state: $\gamma_k = \gamma \sum_m U_{mk} (1 - N_{m,-}) + V_{mk} N_{m,+}$.

Laser cooling drives average occupation numbers $N_{m,\pm}$ of quasiparticle states out of thermal equilibrium. At the same time interactions between quasiparticle states drive the system towards thermal equilibrium. Numerical simulations in Ref. [9] show that for cooling rates in ms range thermal relaxation remains faster than cooling all the way down to $T \simeq 0.03 T_F$. This justifies our assumption of fast equilibration to a quasi-equilibrium state. After each short period of laser cooling the occupation numbers $N_{m,\pm}$ go out of equilibrium, where $N_m = [1 + \exp(\beta\omega_m)]^{-1}$. The initial total energy $E(T) = \sum_{m,\pm} \omega_m N_{m,\pm}$ changes by $dE = \sum_m \omega_m (N_{m,+} + N_{m,-} - 2N_m)$. The relaxation brings the occupation numbers $N_{m,\pm}$ to a new state of equilibrium at a temperature $T + dT$, but it does not change the total energy. The energy of the system at the new temperature $E(T + dT)$ differs only slightly from the initial $E(T)$ by $dE_T = E(T + dT) - E(T) \approx \frac{2dT}{T^2} \sum_m \omega_m^2 N_m (1 - N_m)$. Conservation of energy in thermal relaxation means that $dE = dE_T$. In our simulations we use this equality to find the new lower temperature $T + dT$, and then solve the Bogoliubov-de Gennes equations self-consistently to adjust the Bogoliubov modes and the chemical potential to the lower temperature. With the new Bogoliubov modes we calculate new transition rates etc.

Calculation of the transition rates is the most time consuming part of the numerical simulation. This is why we were forced to assume spherical symmetry. For spherically symmetric $\Delta(r)$ and $W(r)$ the Bogoliubov modes can be separated as e.g. $u_{ln}(r) Y_{lm}(\theta, \varphi)$ and their energies ω_{ln} do not depend on m . To be consistent with the

spherical symmetry we also made the ergodic approximation [9,10], assuming that quasiparticle occupation numbers $N_{ln,\pm}$ do not depend on m . In other words, we assume fast equilibration within each quasiparticle energy shell, which keeps the system in a spherically symmetric state.

In our simulations we use 81 harmonic oscillator levels (shells) and assume that there are $N = 10660$ atoms with spin up and down. In the non-interacting case at $T = 0$ the atoms fill 39 energy shells. The frequency of the isotropic trap is $\omega = 2\pi \cdot 2400\text{Hz}$. The bare scattering length for the interaction between the two species is $a_0 = -157a_B$, where a_B is the Bohr radius. This value corresponds to the interactions between the spin states $|F = 9/2, m_F = 9/2\rangle$ and $|F = 9/2, m_F = 7/2\rangle$ of ^{40}K . In the harmonic oscillator units this scattering length gives $g_0 = -0.32$. In our calculations we use two values for the effective interaction strength: $g = -0.32$ and $g = -1$ enhanced by the Feshbach resonance. The wavelength of the cooling laser is $\lambda = 720\text{nm}$ and the laser detuning is $\delta = -12\hbar\omega$. In all simulations the Rabi frequency is much less than the spontaneous emission rate, $\Omega = 0.1\gamma$, so that average occupation of excited state remains small. This is one of the assumptions in the adiabatic elimination of the excited state.

In Fig.1 we show temperature as a function of time for laser cooling with four values of γ consistent with the *festina lente* regime. Each plot in this figure consists in fact of two indistinguishable plots, one for the bare $g = -0.32$ and the other for the enhanced $g = -1$. For the stronger $g = -1$ there is a transition to the superfluid state at low temperature (see Fig.2), but the non-zero gap function which appears at the transition does not influence the efficiency of laser cooling. The process of laser cooling is dominated by the highly degenerate states with large angular momenta $l \approx E_F/\hbar\omega$ whose quasiparticle energies are not influenced much by the gap function localized in the center of the trap [8,16]. In more physical terms, the superfluid droplet in the center of the trap is surrounded by a shell of normal Fermi gas. As long as this mesoscopic effect persists the laser cooling in the superfluid state remains as efficient as in the normal state.

The initial cooling is the fastest for $\gamma = 10$ but the lowest temperature of $0.085T_F$ is obtained for $\gamma = 1.25$. The best strategy to reach low temperatures in reasonable time is to do the cooling in a few stages starting with large γ and finishing the job with small γ , or to continuously decrease γ as the temperature decreases. The inset in Fig.1 shows an example of a four stage cooling process.

Summarizing, we made numerical simulations for reasonable experimental parameters consistent with the requirements of the *festina lente* regime which demonstrate that it is possible to laser cool fermions on the negative side of the Feshbach resonance all the way down to the Fermi superfluid. Despite the pairing effects and the usual mean field interaction, which shift quasiparti-

cle energies with respect to the equally spaced harmonic oscillator levels, it is experimentally fisible to reach at least $0.085T_F$ in a time as short as a few seconds.

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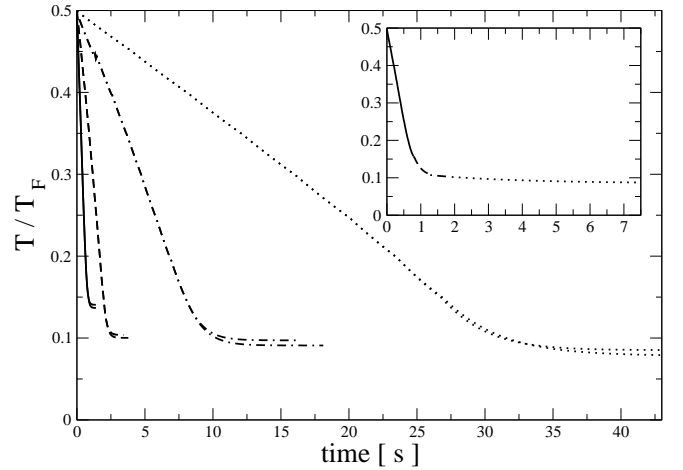


FIG. 1. Temperature as a function of time during the laser cooling for four values of γ : $\gamma = 10$ (solid lines), $\gamma = 5$ (dashed lines), $\gamma = 2.5$ (dashed-dotted lines), and $\gamma = 1.25$ (dotted lines). Each pair of plots in this figure corresponds to $g = -0.32$ (higher temperature) and $g = -1$ (lower temperature). The lowest temperatures of $0.079T_F$ and $0.085T_F$ respectively can be reached for $\gamma = 1.25$. On the other hand, the initial stage of laser cooling is efficient at most for $\gamma = 10$, when the temperature of $0.15T_F$ is reached within 0.85s. The inset shows laser cooling to the superfluid state with $g = -1$ realized in four stages to reach the lowest temperature in reasonable time. The spontaneous emission rate is switched as $\gamma = 10 \rightarrow 5 \rightarrow 2.5 \rightarrow 1.25$ at the temperatures $T/T_F = 0.16, 0.11, 0.10$, correspondingly.

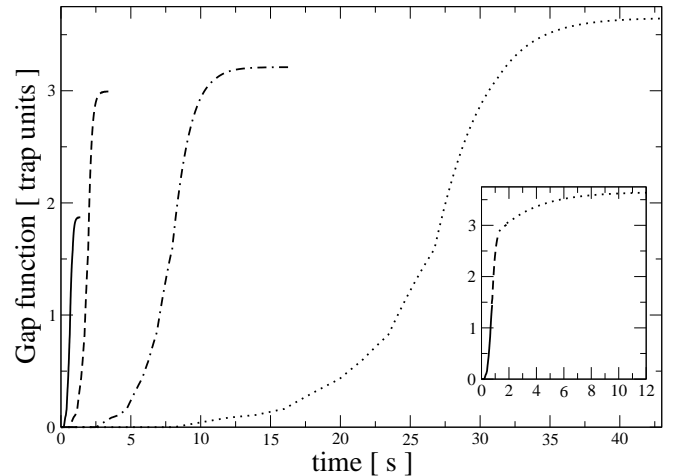


FIG. 2. The maximal magnitude of the gap function in the center of the trap $|\Delta(r = 0)|/\hbar\omega$ for the same simulations with $g = -1$ as in Fig. 1. The inset shows the gap in the four stage cooling of the inset of Fig.1.